Numerical Analysis of Simply Supported Beam on an Elastic Foundation

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Abstract

Multimode vibration analysis of beam type structure has nowadays attained a remarkable attention for various specialized cases. Multimode vibration means when more than one vibration co-exists together. They can intersect each other and resonance occurs, as a result, amplitude becomes more. Multimode free vibration of simply supported beam of different sections rests on elastic foundation is investigated. Numerical methods are analyzed for solving co-exist non-linear system of equations. The first and second mode vibrations are investigated for various moments of inertia of wide-flange sections, equal angle sections, channel sections and different foundation stiffness. In this study, it is noted that the magnitudes of non-linear circular frequencies of equal angle sections are comparatively less than that of other sections. Hence, the equal section is more stable corresponding to large amplitude of vibrations.

***Keywords:* Multimode free vibration, numerical method, vibration amplitude, foundation stiffness.**

1 Introduction

Since many decades, many immense researches about nonlinear beams have been studied– for example, the nonlinear vibrations of nonlinear beams (Ghayesh and Amabili, 2013; Ghayesh and Balar, 2008 and Huang et al., 2011). The nonlinearity of the beam may occur for the mid plane stretching and the large deflection. In the researches (Amabili, 2008), the nonlinear vibrations of flexible beams were investigated and expressed by a set of nonlinear partial differential equations in space and time for different boundary conditions. The partial differential equations were formulated into a set of ordinary differential equations with quadratic and cubic nonlinear terms. It was great time waste for obtaining analytical solutions of such nonlinear vibrations.

The vibration of a beam is an important issue in structural engineering. Various nonlinear beam vibration problems have been investigated and carried out into attentions for example, Hou et al., 2017. These vibration problems are set into model in mathematically as a set of nonlinear partial differential equations. It is generally very difficult to find out the exact solutions of these nonlinear differential equations. Therefore, Numerical methods such as Duan, 2008 are given more importance to find out proper solutions as well as analytical methods (Lai and Yu, 2002). Actually, the authors get concept of governing equation of simply supported beam from researches (Hasan et al., 2016), where an efficient analytical solution method, namely, multi-level residue harmonic balance, is represented. In another researches (Rahman and Lee, 2017) that reflects a new modified multi-level residue harmonic balance method and it is studied out to investigate and analyze the forced nonlinear vibrations of axially loaded double beams. The objectives of the present work are to analyze the multi-mode free vibration and the effect of nonlinear circular frequency for different foundation stiffness on simply supported beam besides comparison of nonlinear circular frequency of simply supported beam for different moment of inertia for different sections i.e., wide flange sections, equal angle sections and channel sections.

2 Theoretical models

The governing equation of motion for nonlinear vibrations of simply supported beam showing the effect for mid plane stretch based on Euler-Bernoulli theory is as follows:

$$EIW'''' + \rho A\ddot{W} + \kappa_f W - \left(\frac{EA}{2L}\int_0^L (W')^2 dx\right) W'' = 0 \quad (1)$$

where, x represents the longitudinal coordinate of the beam, W is the transverse displacement for mid plane stretching with respect to the longitudinal coordinate of the beam. W', W'', and W'''' are the 1st, 2nd, and 4th derivatives with respect to x, respectively; besides $\ddot{W}$ is the 2nd derivative with respect to time and t denotes time. L shows beam length and the cross-sectional area of the beam, $A = B$ X h, B and h represents the beam width and thickness respectively as well as the properties of the beam including the Young's modulus of the beam E, the material density ρ, foundation coefficient of modulus κ_f. The relevant necessary boundary conditions for simply supported beams are

$$W(x,t) = W''(x,t) = 0, \qquad at\ x = 0, L \quad (2a)$$
$$W(x,t) = W'(x,t) = 0, \qquad at\ x = 0 \quad (2b)$$
$$W(x,t) = W''(x,t) = 0, \qquad at\ x = L \quad (2c)$$

The governing equation is investigated by successful application of the Galerkin procedure. The Transverse displacement is represented in means of beam mode shapes:

$$W(x,t) = \sum_{i=1}^{n} \mathrm{q}_i(t)\varphi_i(x) \quad (3)$$

where, q_{i} = the modal amplitude for the ith mode, φ_i represents ith structural mode shape. For simply supported beam

$$\varphi_i(x) = \sin\left(\frac{\mathrm{n}\pi\mathrm{x}}{\mathrm{L}}\right) \quad (4)$$

where, i = the structural mode number and n = the number of structural modes. Hence, the residual can be defined by substituting equation (3) into equation (1) as follows:

$$\Delta = EI\sum_{i=1}^{n} \mathrm{q}_i\varphi_i'''' + \rho A\sum_{i=1}^{n} \ddot{\mathrm{q}}_i\varphi_i + \sum_{i=1}^{n} \kappa_f \mathrm{q}_i\varphi_i - \frac{EA}{2L}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} \mathrm{q}_i q_j q_k\varphi_i''\int_0^L \varphi_i'\varphi_j' dx \quad (5)$$

where φ_i', φ_i'', φ_i'''' show the first, second, and fourth derivatives of the i-th mode shape; $\ddot{\mathrm{q}}_i$ is 2nd derivatives of the modal amplitude of the i-th mode with respect to time; and i, j and k are the mode numbers. Equation (5) contains cubic nonlinear term. According to the Galerkin approach, the weighted residual in equation (5) is set to zero. Multiplying φ_m each term on the right-hand side of equation (5), and taking integration over the beam length,

$$\rho A\sum_{i=1}^{n} \ddot{\mathrm{q}}_i\,\alpha_{i,m}^{0.0} + EI\sum_{i=1}^{n} \mathrm{q}_i\,\alpha_{i,m}^{4.0} + \sum_{i=1}^{n} \kappa_f \mathrm{q}_i\,\alpha_{i,m}^{0.0} - \frac{EA}{2L}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} \mathrm{q}_i q_j q_k\alpha_{i,m}^{2.0}\,\alpha_{i,j}^{1.1} = 0 \quad (6)$$

where,

$$\alpha_{i,m}^{0.0} = \int_0^L \varphi_i\varphi_m dx, \quad \alpha_{i,m}^{1,1} = \int_0^L \varphi_i'\varphi_m' dx, \quad \alpha_{i,m}^{2.0} = \int_0^L \varphi_i''\varphi_m\, dx, \quad \alpha_{i,m}^{4.0} = \int_0^L \varphi_i''''\varphi_m\, dx$$

For Multi-mode residue harmonic balance, considering a two mode approach:

$$W(x,t) = \mathrm{q}_0(t)\varphi_1(x) + \mathrm{q}_2(t)\varphi_3(x) \quad (7)$$

The following coupled nonlinear ordinary differential equations are obtained respectively:

$$\ddot{\mathrm{q}}_1 + \omega_1^2 q_0(1+\kappa_1) + \alpha_{11}q_0^3 + \alpha_{13}q_0^2q_2 + \alpha_{31}q_0q_2^2 + \alpha_{33}q_2^3 = 0 \quad (8a)$$
$$\ddot{\mathrm{q}}_3 + \omega_3^2 q_2(1+\kappa_3) + \beta_{11}q_0^3 + \beta_{13}q_0^2q_3 + \beta_{31}q_0q_3^2 + \beta_{33}q_2^3 = 0 \quad (8b)$$

3 Numerical methods (Runge-Kutta method)

It is one of the most widely used methods and it is particularly suitable in case when the computation of higher derivative is complicated. The Runge-Kutta methods are designed to give greater accuracy and they possess the advantage of requiring only the function values at some selected points on the subinterval From Modified Euler's Method

$$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)] \quad (9)$$

Now, substitute $y_1 = y_0 + hf(x_0, y_0)$on the right side of equation (9)

$$y_1 = y_0 + \frac{h}{2}[f_0 + f(x_0 + h, y_0 + hf_0)] \tag{10}$$

where, $f_0 = f(x_0, y_0)$. If we now set

$$k_1 = hf_0 \text{ And } k_2 = hf(x_0 + h, y_0 + k_1) \tag{11}$$

Then the above equation becomes

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2) \tag{12}$$

Which is the second order Runge-Kutta formula? The error in this formula can be shown to be of order h^3 by expanding both sides by Taylor's series. Thus the left side gives

$$y^0 + hy'_0 + \frac{h^2}{2}y''_0 + \frac{h^3}{6}y'''_0 + \cdots \tag{13}$$

And on the right side

$$k_2 = hf(x_0 + h, y_0 + hf_0) = \mathrm{h}\left[f_0 + h\frac{\partial f}{\partial x_0} + hf_0\frac{\partial f}{\partial y_0} + 0(h^2)\right] \tag{14}$$

Since $\frac{df(x,y)}{dx} = \frac{\partial f}{\partial x} + f\frac{\partial f}{\partial y}$, We obtain

$$k_2 = h[f_0 + hf'_0 + 0(h^2)] = \mathrm{h}f_0 + h^2f'_0 + 0(h^3) \tag{15}$$

So that the right side of (12) gives

$$y_0 + \frac{1}{2}\left[hf_0 + hf_0 + h^2f'_0 + 0(h^3)\right] = y_0 + hf_0 + \frac{1}{2}h^2f'_0 + 0(h^3)$$
$$= y_0 + hy'_0 + \frac{h^2}{2}y''_0 + 0(h^3) \tag{16}$$

If therefore follows that the Taylor series expansions of both sides of (12) agree up to terms of order h^2, which means that the error in this formula is of order h^3.More generally, if we set

$$y_1 = y_0 + w_1k_1 + w_2k_2 \tag{17}$$

Where
$k_1 = hf_0$
$k_2 = hf(x_0 + \alpha_0h, y_0 + \beta_0k_1)$
Then the Taylor series expansions of both sides of the last equation in (17) give the identity

$$y_0 + hf_0 + \frac{h^2}{2}\left(\frac{\partial f}{\partial x} + f_0\frac{\partial f}{\partial y}\right) + 0(h^3) = y_0 + (w_1 + w_2)hf_0 + w_2h^2\left(\alpha_0\frac{\partial f}{\partial x} + \beta_0f_0\frac{\partial f}{\partial y}\right) + 0(h^3) \tag{18}$$

Equating the coefficients of $f(x_0, y_0)$ and its derivatives on both sides, we obtain the relations

$$\left.\begin{aligned} w_1 + w_2 &= 1 \\ w_2\alpha_0 &= \frac{1}{2} \\ w_2\beta_0 &= \frac{1}{2} \end{aligned}\right\} \tag{19}$$

Clearly $\alpha_0 = \beta_0$ and if α_0 is assigned any value arbitrarily, then the remaining parameters can be determined uniquely, If we set for example,$\alpha_0 = \beta_0 = 1$ then we immediately obtain $w_1 = w_2 = 1/2$ which gives formula (3.4).
Higher-order Runge-Kutta formulae exist, of which we mention only the fourth-order formula defined by

$$y_1 = y_0 + w_1k_1 + w_2k_2 + w_3k_3 + w_4k_4 \tag{20}$$

where

$$\left.\begin{aligned} k_1 &= hf(x_0, y_0) \\ k_2 &= hf(x_0 + \alpha_0 h, y_0 + \beta_0 k_1) \\ k_3 &= hf(x_0 + \alpha_1 h, y_0 + \beta_1 k_1 + v_1 k_2) \\ k_4 &= hf(x_0 + \alpha_2 h, y_0 + \beta_2 k_1 + v_2 k_2 + \delta_1 k_3) \end{aligned}\right\}$$

The choice of the parameters is arbitrary and we have therefore several fourth-order Runge-Kutta formulae. The authors are utilized this method to analysis the non-linear beam vibrations by the Mathcad Software.

.4 Results and discussion

In this section, the material properties of the beams in the numerical cases are considered as follows: Young's modulus of the beam $E = 210 \times 10^9$ N/m^2, mass density ρ = 7800 kg/m^3. There is one boundary condition considered for simply supported beam. Figure 1 plots the non-linear circular frequencies corresponding to vibration amplitude for various foundation coefficient of modulus. It can be seen that with the increase of foundation coefficient of modulus the non-linear circular frequencies decrease for the both cases of symmetric and anti-symmetric mode of vibration. The non-linear circular frequencies are plotted against various initial amplitudes for 1st and 2nd modes of vibrations in Figure 2. Figure 2 depicts that, with the increase of amplitude the vibration also increase. Figures 3, 4 and 5 plot the non-linear circular frequencies with respect to various initial amplitudes for 1st and 2nd modes of vibrations for the different magnitude of moment of inertia of wide flange sections, equal angle sections and channel sections respectively. It can be seen that with the increase of moment of inertia for that three sections the non-linear circular frequencies increase. Figures 3, 4 and 5 depict that the non-linear circular frequencies are less for equal angle sections than that of other sections.

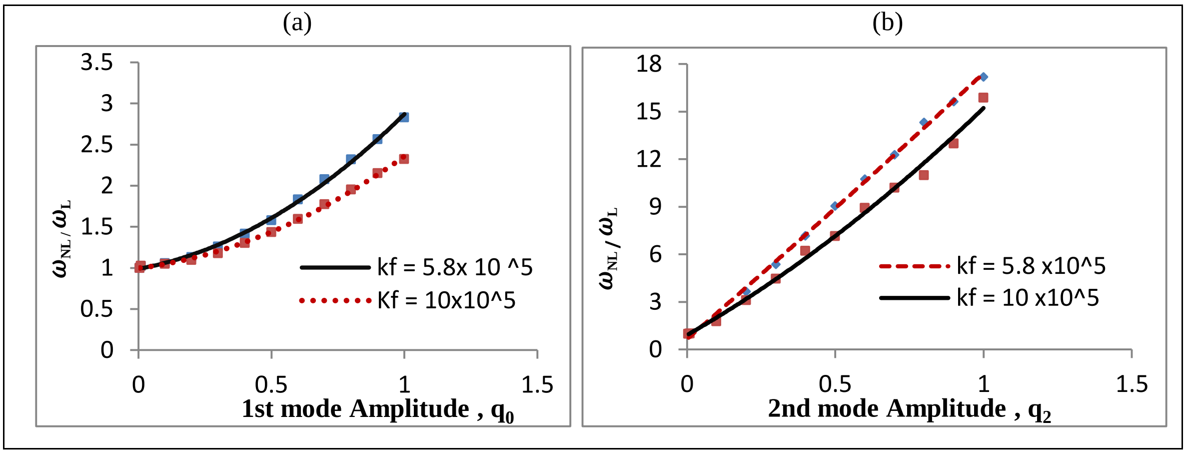

Figure 1. (a) 1st mode and (b) 2nd mode vibrations corresponding to amplitude for various foundation stiffness.

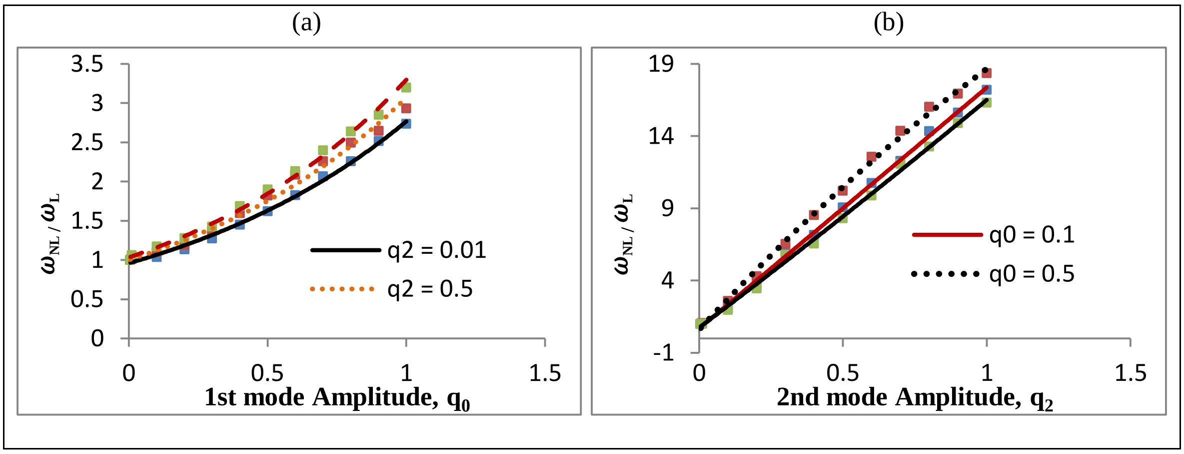

Figure 2. (a) 1st mode and (b) 2nd mode non-linear circular frequencies corresponding to different initial amplitudes.

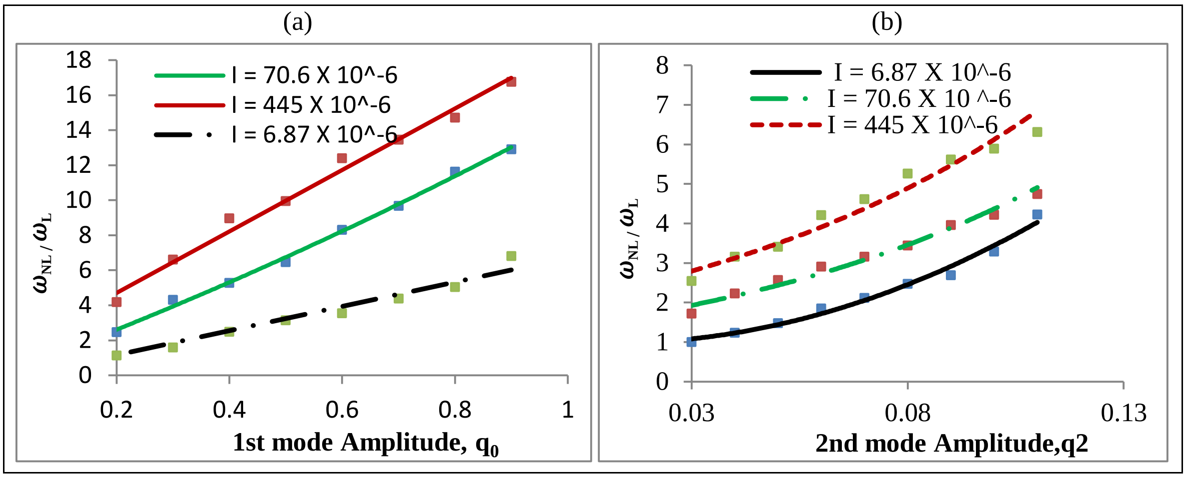

Figure 3. (a) 1st mode and (b) 2nd non -linear frequencies corresponding to various moments of inertia for wide flange section of simply supported beam.

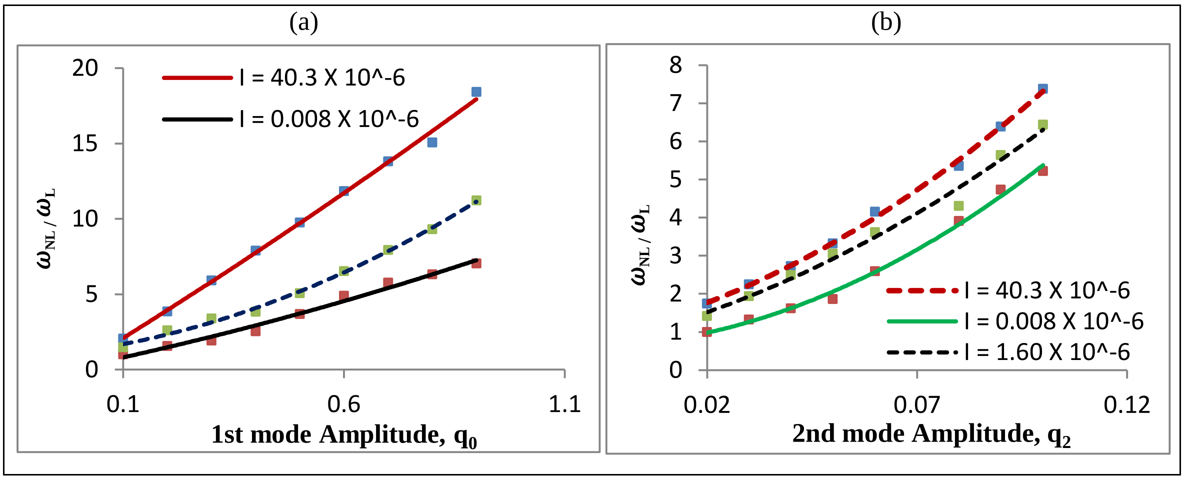

Figure 4. (a) 1st mode and (b) 2nd non-linear circular frequencies corresponding to various moments of inertia for equal angle section of simply supported beam.

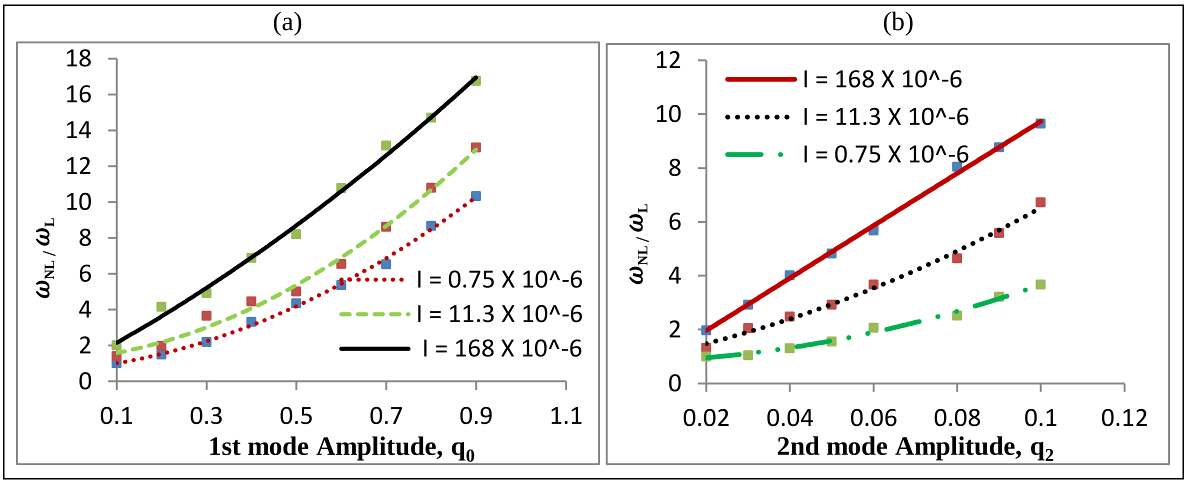

Figure 5. (a) 1st mode and (b) 2nd non-linear frequencies corresponding to various moments of inertia for channel section of simply supported beam.

5 Conclusions

In this study, numerical integration method namely, Runge-Kutta method is successfully employed for non-linear simply supported beam on elastic foundation. Non-linear circular frequencies are investigated for wide flange sections, equal angle sections and channel sections. The non-linear circular frequencies are less for equal angle sections than that of other sections. Therefore, it can be concluded that the equal angle sections are more stable corresponding to the larger amplitude of vibration.

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