# Development of a Mathematical Model for Unsteady Flow in a Single Channel of the Karnaphuli River for Dry and Wet Season

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**Abstract**

Chittagong, the port city of Bangladesh is situated on the bank of the Karnaphuli River between the Chittagong hill tracts and the Bay of Bengal. Furthermore, Karnaphuli Hydropower Station is the only hydropower plant in the country. It is located at kaptai as a part of the "Karnaphuli Multipurpose Project". In this circumstance, it is indispensible to predict the flow characteristics of the river in different periods to take necessary steps at vital time. For this reason, a one-dimensional model to predict tides in a single channel has been developed and applied to the Karnaphuli River starting from the sea face at Patenga to Kaptai dam. This model comprises of the equations of continuity and momentum for unsteady gradually varied flow. The 72.95 km long river from Patenga to Kaptai consists of 16 grid points at 4.86 km interval and river sections have been assumed as rectangular sections. The model has been run for a semidiurnal tide having a tidal period of 44760 seconds (12 hours 26 minutes) and the total tidal period has been divided into 40 time intervals of 4119 seconds. To construct the model a C++ computer program has been written and the model is capable of predicting the water levels (stages), mean flow velocities and discharge at different sections for dry and wet season.

***Keywords:*** *Karnaphuli River, prediction, tidal flow, unsteady gradually varied flow, dry and wet Season.*

## 1 Introduction

A model is simplified representation of a complex system or process or phenomena. There are two types of models, physical model and mathematical model. A physical model is used if the physical phenomenon can be reproduced with sufficient similarity by reducing the length dimensions of the real system. This requires much space, time and money. A mathematical model is a simplified representation of a complex system in which the behavior of the system is represented by a set of equations, based on the fundamental principles of mass, energy and momentum conservation. Mathematical modeling in rivers is based on the formulation and solution of mathematical relationships of known principles of hydraulics. The behavior of river Karnaphuli has great influence upon the functioning of the Chittagong port. One of the main problems confronting the port authority has been to maintain and improve the navigability of the channel alongside the jetties and stabilize the riverbanks. Therefore, to study the navigability, sediment transport and other factors in the river Karnaphuli is of great importance in order to maintain sufficient depth in the river. The objective of the study is to develop a one-dimensional mathematical model for unsteady gradually varied flow in a single channel during dry and wet season.

## 2 Study Area

The Karnaphuli is the largest and most important river in Chittagong and the Chittagong Hill Tracts, is a 667 m (2,188 feet) wide river in the south-eastern part of Bangladesh. Originating from the Lushai hills in Mizoram, India, it flows 270 kilometers (170 miles) southwest through Chittagong Hill Tracts and Chittagong into the Bay of Bengal. A large hydroelectric power plant using Karnaphuli River was built in the Kaptai region during the 1960s. The mouth of the river hosts Chittagong sea port, the main port of Bangladesh (Karnafuli, n.d.).

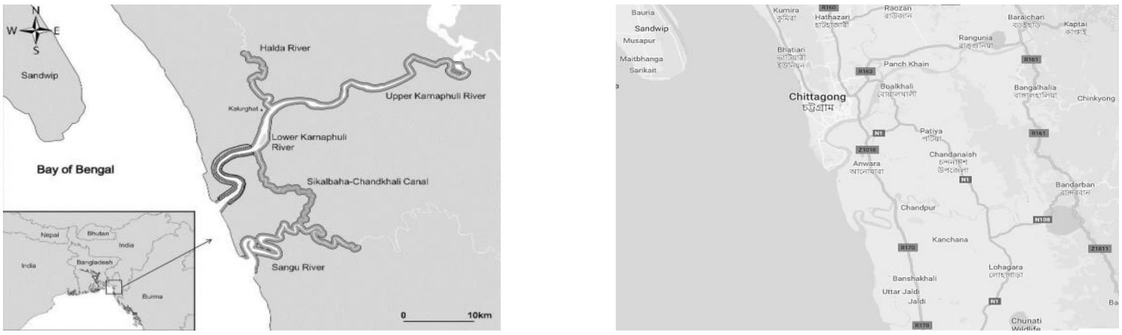


Figure 1. A plan view and google earth view of the Karnaphuli River

## 3 Data Collection

### 3.1 Methodology

Different model components could be exemplified by the following equation which describes the flow through a homogeneous and isotropic confined aquifer with constant transmissivity, T and external recharge, R(x,y):

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{R\,(x,y)}{T} \tag{1.1}$$

This equation is also known as the Poisson equation of steady state confined ground water flow (Wang and Anderson, 1982).The variables in this case are x, y and h. The first two are independent variables and the latter one is a dependent variable. The parameters in this model are T and R(x, y). Note that although R(x, y) is a function of space, it remains constant for a particular location. The mathematical model considered in this study consists of the Saint Venant equations, the equation of continuity and the equation of momentum. The Saint Venant equations are written as:

$$\frac{\partial Q}{\partial x} + B\frac{\partial H}{\partial t} - q = 0 \tag{1.2}$$

$$\frac{\partial Q}{\partial t} + \frac{2Q}{A}\frac{\partial Q}{\partial x} - \frac{Q^2}{A^2}\frac{\partial A}{\partial x} + gA\frac{\partial H}{\partial x} + g\frac{n^2|Q|Q}{AR^{\frac{4}{3}}} = 0 \tag{1.3}$$

where Q is the discharge, H is the stage, q is the lateral flow, B is the top width, A is the cross-sectional rea of flow, g is the acceleration due to gravity, R is the hydraulic radius, x is the space co-ordinate, t= time coordinate During the flood periods, the top width B is equal to the sum of the widths of left berm, main channel and right berm. During the flood periods, the governing equations consider only the flow velocity that exists in the main channel and only with storage effect in the berms.

The continuity and momentum equations are converted to linear differential equations. The finite differential equations are written for every grid from upstream to downstream. The total numbers of equation are equal to the number of grids. At each boundary of the model the discharge or water level hydrograph must be given. The unknowns are then solved simultaneously using the double sweep method. It is assumed the stage and discharge are zero at the beginning of computation. Due to the hyperbolic nature of the equations, the model converges to the true solution after a few iterations. The model requires water levels (stages) at different time levels at the downstream boundary and discharges at different time levels at the upstream boundary. However, in the present study, a constant discharge at Kaptai has been used. Finally, a computer program has been written in C++ to compare the water levels, velocities and discharges. From the model, Quasi-steady condition obtained after 6 no. of iterations and has found Results of Variation of water level, discharge and velocity along the length of the channel.

### 3.2 Longitudinal and Cross-Sectional Schematization

The model is applied to 72.95 km long lower Karnaphuli River from Patenga to Kaptai. The longitudinal schematization consists of 16 sections with a space interval of 4.86 km. The cross-sections are represented by equivalent rectangular sections keeping this water surface width constant. The model has been run for dry season water level data at Patenga sea face and a constant discharge of 1000 $m^3$/s at Kalurghat (released from Karnaphuli power station at Kaptai) and for a semi diurnal tide having a tidal period of 12 hours 26 minutes with 40 time intervals (time interval t=1119 seconds=18.65 minutes). The Mannings roughness coefficient n=0.025 has been used. The model is capable to compute the water levels and discharges at all the cross-sections at various time levels statistically (Halim, 2006) which is shown in table 1 and 2 respectively.

Table 1. Water table data used in the model

| Time level | Water level (Dry season) | Water level (Wet season) |
|---|---|---|
| 1 | 4.30 | 6.54 |
| 2 | 5.10 | 7.75 |
| 3 | 5.90 | 8.97 |
| 4 | 6.50 | 9.88 |
| 5 | 7.00 | 10.64 |
| 6 | 7.30 | 10.10 |
| 7 | 7.40 | 11.25 |
| 8 | 7.10 | 10.79 |
| 9 | 6.60 | 10.03 |
| 10 | 5.90 | 8.97 |
| 11 | 5.00 | 7.60 |
| 12 | 4.10 | 6.23 |
| 13 | 3.10 | 4.71 |
| 14 | 2.10 | 3.19 |
| 15 | 1.20 | 1.82 |
| 16 | 0.20 | 0.30 |
| 17 | - 0.80 | - 1.22 |
| 18 | - 1.80 | - 2.74 |
| 19 | - 2.70 | - 4.10 |
| 20 | - 3.50 | - 5.32 |

Table 2. Cross-section data used in the model

| Section | Depth (m) | Width (m) |
|---|---|---|
| 1 | 9.54 | 685.98 |
| 2 | 8.73 | 533.54 |
| 3 | 9.97 | 381.1 |
| 4 | 6.44 | 737.8 |
| 5 | 4.10 | 975.61 |
| 6 | 3.71 | 1105.18 |
| 7 | 3.29 | 472.56 |
| 8 | 4.32 | 304.88 |
| 9 | 1.92 | 472.56 |
| 10 | 1.78 | 304.88 |
| 11 | 0.79 | 449.7 |
| 12 | 0.92 | 304.88 |
| 13 | 1.05 | 312.5 |
| 14 | 1.46 | 213.41 |
| 15 | 2.13 | 198.17 |
| 16 | 1.46 | 213.41 |

It was assumed that the stage and discharge is zero at the beginning of computation. Due to the hyperbolic nature of the equations, the model converges to the true solution after a few iterations. The model requires water levels (stages) at different time levels at the downstream boundary and discharges at different time levels at the upstream boundary. However, in the present study, a constant discharge at Kaptai has been used. A computer program has been written in C++ to compare the water levels, velocities and discharges in dry and wet season using the model.

## 4 Model Results

The mathematical model has been run for the dry season water level (stage) data at the downstream boundary Patenga for the semidiurnal tide of 12 hours and 26 minutes with $\Delta x$= 4863.33 inches and $\Delta t$= 1119 seconds and a discharge of 1000 $m^3/s$ at the upstream boundary Kaptai. The numerical value of the roughness coefficient n has been taken as 0.025 (Halim, 2006). The model results include the water levels (stages), mean flow velocities and discharge at all the 16 cross-sections at different time levels. The predicted results are exactly same as those predicted by Halim (2006). The model thus seems to be capable of predicting water levels and discharges at all the channel sections. In the figures 2 to 10 the predicted water levels, discharges, and velocities at section 5, 10 and 15 have been shown.

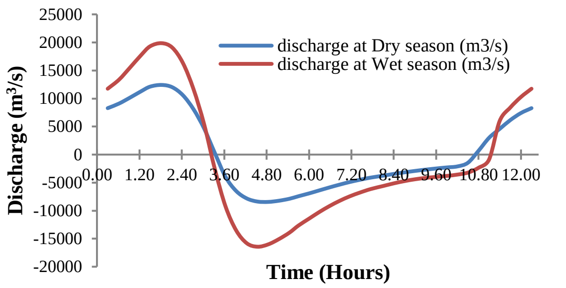


Figure 2. Variation of discharge for dry and wet season at Section 5

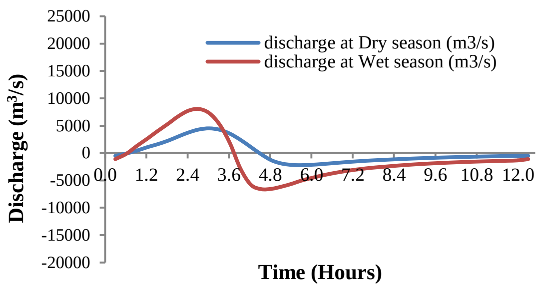


Figure 3. Variation of discharge for dry and wet season at Section 10

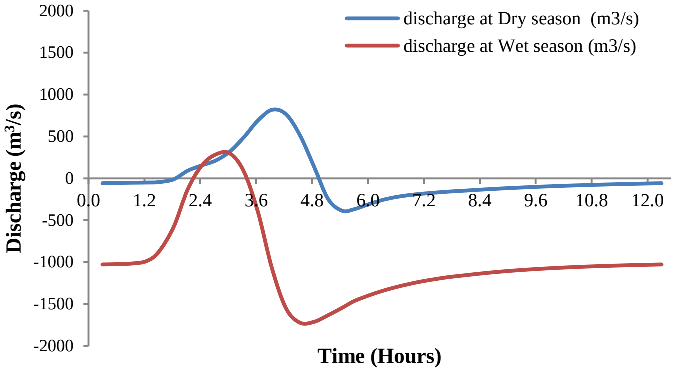


Figure 4. Variation of discharge for dry and wet season at Section 15

From the figures 2, 3 and 4, it is clear that the increase or decrease of discharge in the upstream of the river is higher than the downstream of the river. In dry season discharge at section 5 and 10 are 12425.13 $m^3/s$ and 4306.96 $m^3/s$ respectively & for wet season the discharge are found to be 19872.99 $m^3/s$ and 7744.15 $m^3/s$, where at the end portion of the river (section 15) the discharge is 762.97 $m^3/s$ in dry season & 296.49 $m^3/s$ in wet season.

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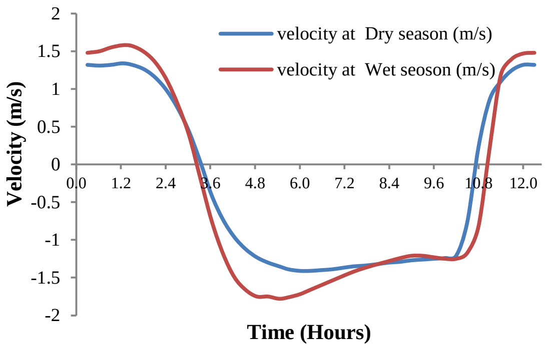


Figure 5. Variation of velocity for dry and wet season at Section 5

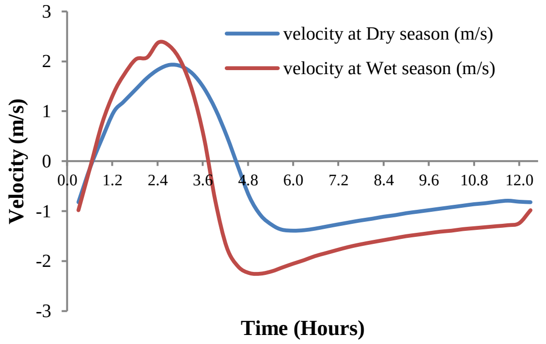


Figure 6. Variation of velocity for dry and wet season at Section 10

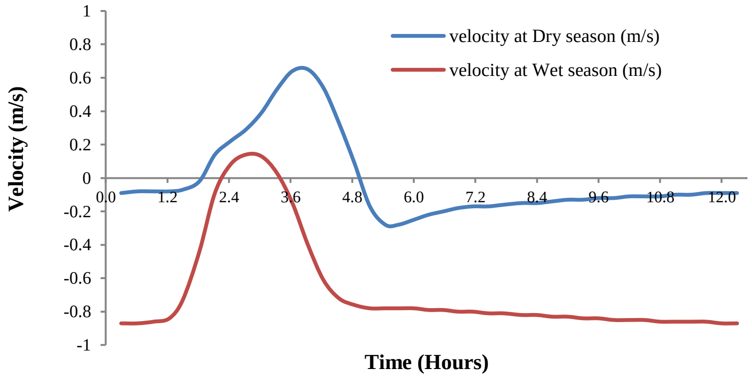


Figure 7. Variation of velocity for dry and wet season at Section 15

From the figures 5, 6 and 7, it is clear that the increase or decrease of velocity in the upstream of the river is higher than the downstream of the river. In dry season velocity at section 5 and 10 are 1.48 m/s and 1.98 m/s respectively & for wet season the velocity are found to be 1.57 m/s and 2.38 m/s, where at the end portion of the river (section 15) the velocity is 0.68 m/s in dry season & 0.4 m/s in wet season.

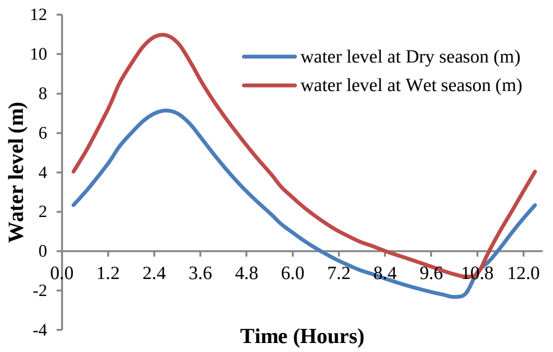


Figure 8. Variation of water level for dry and wet season at Section 5

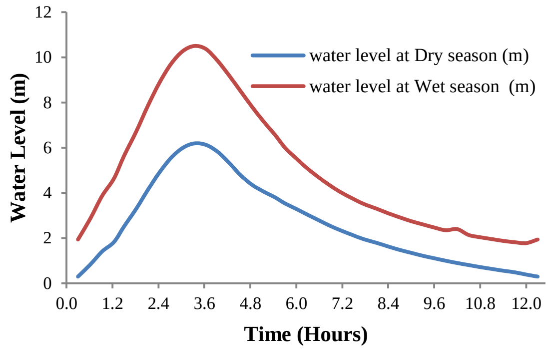


Figure 9. Variation of water level for dry and wet season at Section 10

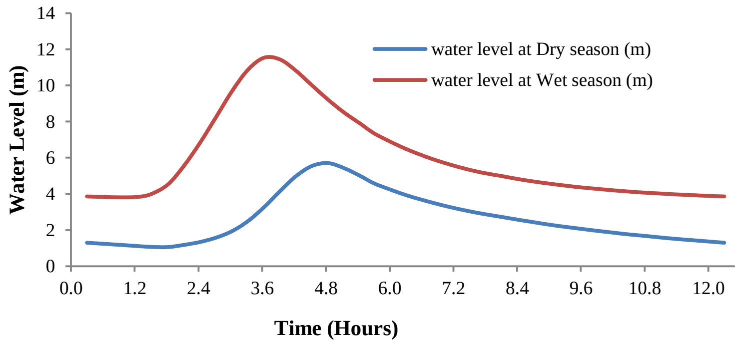


Figure 10. Variation of water level for dry and wet season at Section 15

From the figures 8, 9 and 10, it is clear that the increase or decrease of water level (stages) in the upstream of the river is more than the downstream of the river. In dry season water level at section 5 and 10 are 7.15 m and 5.99 m respectively & for wet season the water level are found to be 10.95 m and 10.5 m, where at the end portion of the river (section 15) the water level is 5.55 m in dry season & 11.53 m in wet season.

The results depict, The Karnaphuli River have much higher discharge, velocity and water level in both Dry and Wet season at the upstream than the downstream. It may happen due to the river are carrying more water, the variation of width and carrying of the bulk amount of sediments to downstream as depth is decreasing to downstream, etc. An increase in upstream discharge at Kaptai increases the ebb flow and ebb flow velocity but decreases the flood flow and flood flow velocity at all the channel sections. However, an increase in Kaptai discharge causes an increase in water levels at all the channel sections. The increase or decrease in discharge, velocity and water level is more in the upstream sections than in the downstream sections.

## 5 Conclusion

The one dimensional mathematical model developed for unsteady gradually varied flow can be used to predict the water levels (stages), velocities and discharges in the Karnaphuli river at all the river sections and at different time levels. In addition, the increase in downstream water levels at Patenga increases the water levels, discharges and flow velocities at all the river sections. Lastly, the model developed for one-dimensional unsteady flow can be used for other tidal rivers of Bangladesh.

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