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Experimental Investigation of Beams on Elastic Foundation Subjected to Static Concentrated Load

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Abstract

In civil engineering, there are countless situations where a flexible distributed foundation supports a relatively rigid structure. A common and significant example is when buildings, railroads, or other structures are supported on soft soil bases, and prismatic beams often experience elastic deformation when resting continuously on such foundations. The bending of beams on an elastic foundation is developed on the assumption that the reaction forces on the elastic foundation are linearly proportional to the beam deflection at any point. The soil mass beneath the foundation is represented as a set of closely separated, identical, and independent linear elastic springs, known as the Winkler model. This simplification allows for easier analysis of problems and is commonly used as an approximation in various situations. This paper investigates the behavior of beams on elastic foundations under static concentrated loads analytically and experimentally. The experimental project involves placing unconnected springs along the length of prismatic beams with varying cross-sectional dimensions and measuring the deflection at different points when a static point load is applied at the middle. The experimental values at different sections are compared with the theoretical value derived from the analytical solution, highlighting the tendency of theoretical models to overestimate deflection. It shows that larger cross-sectional areas lead to better alignment between actual and theoretical deflection. The theoretical model accurately identifies zero deflection points, affirming its validity.

Keywords: *Elastic foundation, Prismatic beam, Flexible Springs, Experimental model, Deflection.*

1 Introduction

The application of beams on elastic foundations is widely observed in mechanical and civil engineering, such as disc brake pads, shafts supported on various bearings, vibrating machines on elastic foundations, floor systems for ships, buildings, and bridges, submerged floating tunnels, buried pipelines, and railroad tracks. The computational model of a beam or plate on an elastic foundation is widely used in engineering fields such as geotechnics, road, railroad, marine engineering, and bio-mechanics. The key challenge is modeling the contact between the beam and the soil bed. Typically, spring elements are used to represent the contact, prioritizing beam analysis over soil behavior. The stiffness of these springs determines the foundation's behavior, and various methods exist to determine their values. In a linear elastic foundation, displacement is linearly related to the applied load. The Winkler foundation model treats the soil as a series of closely spaced, independent, linearly elastic springs. However, it has a limitation of a displacement discontinuity that does not reflect real soil behavior. Despite this, the computational model provides valuable insights into engineering problems, enabling the analysis of structures in different applications. Researchers have extensively studied this area, employing analytical, numerical, and experimental approaches.

In 1937, Biot et al. discussed the bending of an infinite beam on an elastic foundation. They characterized the foundation as an elastic continuum with two elastic constants: modulus of elasticity (E) and Poisson's ratio (v). The authors provided a more precise solution for the case of an infinite beam under a concentrated load, considering both two-dimensional and three-dimensional foundations. Hetenyi and Hetbenyi (1946, 1961) and Timoshenko (1956) further explored the application of beams on elastic foundations, presenting analytical solutions based on classical differential equation approaches and considering various loading and boundary conditions. Ellington (1957) investigated the conditions under which a beam supported by discrete elastic supports can be treated as equivalent to a beam on an elastic foundation. The appropriateness of this analogy depends on the flexural rigidity of the beam and the stiffness and spacing of the supports.

Winkler et al. (1967) introduced a simple mechanical representation of soil foundation using discrete and linearly elastic springs. However, the Winkler model has limitations in accurately representing practical foundation soils. It neglects the interaction between adjacent springs and the vertical shearing stress within subgrade materials. Additionally, it assumes a displacement discontinuity on the foundation surface, which is not reflective of reality. Teodoru and Mușat (2010) proposed the modified Vlasov foundation model as an alternative to the Winkler model. This approach provides a more comprehensive understanding of stress and deformation within the soil mass. By applying the Vlasov approach to static analysis, the need for arbitrary determination of foundation parameters is eliminated. Dinev (2012) presented an analytical solution for beams on elastic foundations using singularity functions. The author introduced a new approach based on variational formulation and the minimum potential energy functional. This method offers advantages in solving the equilibrium equation and applying boundary conditions. Boudaa et al. (2019) conducted a static interaction analysis between a beam and layered soil using a two-parameter elastic foundation. They employed finite element modeling, considering the linear and homogeneous isotropic behavior of the soil and the beam. The analysis incorporated the strain energy expressions of both components, and the stiffness matrix of each component was integrated into the finite element analysis.

While most research on beams on elastic foundations has focused on analytical and numerical studies, there have been limited experimental investigations. In this study, a practical model using a series of helical springs to represent the elastic foundation has been fabricated. The experimental and theoretical deflection under static point loading has been compared to provide insights into the behavior of beams on elastic foundations.

2 Methodology

2.1 Theoretical Background

The concept of bending beams on an elastic foundation is based on the assumption that the underlying soil mass can be represented by a series of closely spaced linearly elastic springs. This representation, known as the Winkler foundation, assumes that the reaction forces exerted by the foundation are directly proportional to the deflection of the beam at each point. The deflection of an infinite beam due to a point load at mid span has shown in Figure 1.

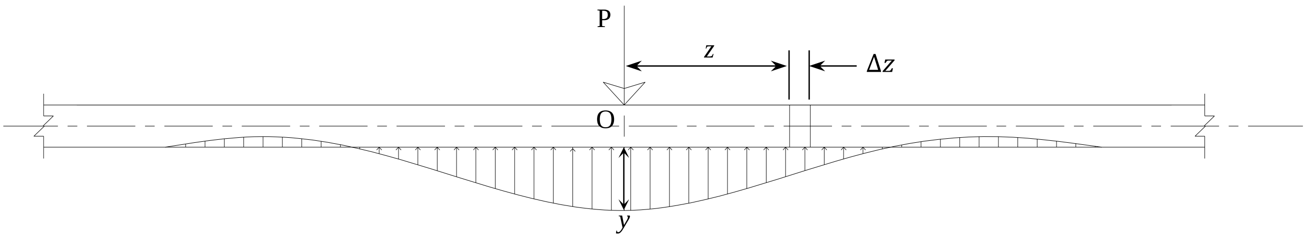

Figure 1. SPT blow counts for the profile investigated.

Governing differential equations for a constant point load at center is given by (J.R. Barber, 2011)-

$$EI\,\frac{d^4y}{dz^4} = -ky$$

Where, E is the Modulus of Elasticity, I is the Moment of Inertia and k is the elastic coefficient or foundation modulus for the model = $\frac{spring\ constant, K}{l}$.

After solving this homogeneous, fourth order, linear differential equation, for Winkler foundation, deflection of infinite beam subjected to point load can be derived as,

$$y = \frac{P\beta}{2k} A_{\beta z}$$

Where, P is point load on mid span , $A_{\beta z} = e^{-\beta Z}(\sin\beta z + \cos\beta z)$ and $\beta = \sqrt[4]{\frac{k}{4EI}}$

2.2 Experimental Program

The experimental model consists of nine helical springs of similar dimensions, each equipped with a guide tube to prevent lateral deflection as shown in Figure 2. A sturdy support structure, including a base beam and three columns, is used to apply the test forces. The base beam is a steel channel section, while the columns are steel hollow pipes. Different loads, ranging from 2.25 kg to 9 kg, are applied at the midspan of the beam using a loading chord. Deflection measurements are taken using a dial gauge with a precision of 0.01 mm. The model allows for the determination of beam deflection at various points under static point loads at the midspan.

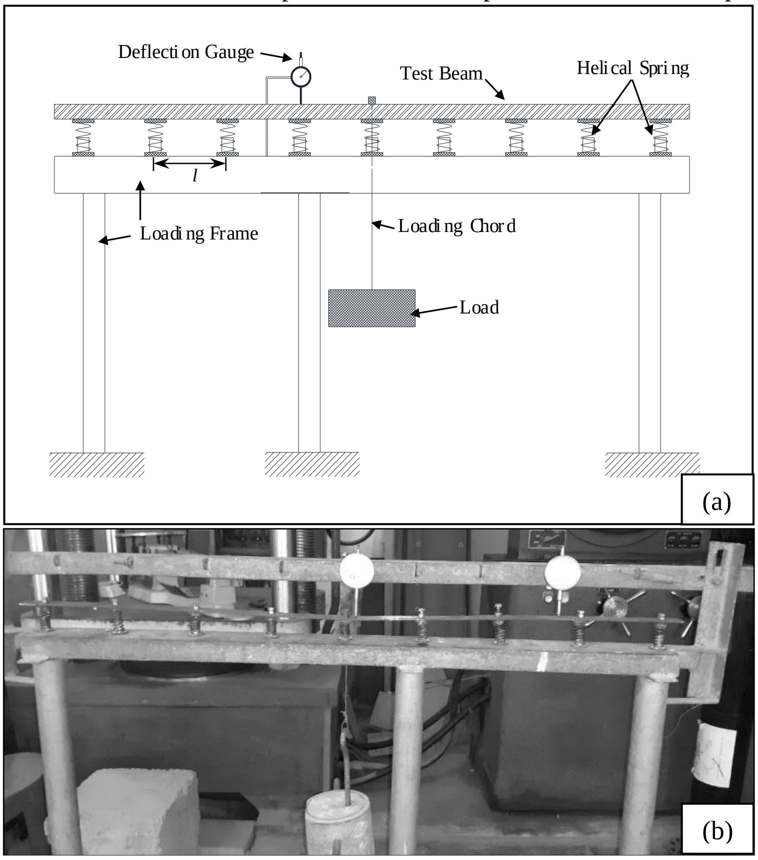

Figure 2. Test set-up for determination of deflection of beam on elastic foundation (a) Schematic diagram (b) Actual setup.

The model is constructed using nine helical springs that have approximately the same dimensions. To prevent lateral deflection, each spring is equipped with a guide tube placed inside it, as depicted in Figure 3. The height of the guide tube is approximately half the height of the spring. The specific details regarding the dimensions and mechanical properties of these springs can be found in Table 1.

Figure 3. Helical spring. (a) Schematic diagram (b) Actual spring.

Rectangular mild steel foils of three distinct cross sections were used as test beams, as illustrated in Figure 4. The dimensions and mechanical properties of the beam specimens are provided in Table 2. When, the beam is defined as a Long beam or infinite beam (J.R. Barber, 2011). Therefore, all three beams in the Table 2 can be considered infinite beams.

Table 1. Dimensions and mechanical properties of springs.

Sample No	Interior Diameter, D_{int} (mm)	Exterior Diameter, D_{ex} (mm)	Height of spring, H (mm)	Diameter of Spring chord, d (mm)	Spring Constant, K (N/mm)	Spacing of Spring, l (mm)	Foundation Modulus, $k=\frac{K}{l}$ (N/mm^2)
Spring 1	12.52	16.96	39.21	1.99	7.495	123.00	0.0602
Spring 2	13.52	17.48	39.10	1.97	7.762	124.50	0.0624
Spring 3	14.52	17.56	39.54	1.92	7.000	125.00	0.0562
Spring 4	15.52	17.40	38.44	1.86	7.243	124.50	0.0582
Spring 5	16.52	17.52	39.60	1.98	7.887	124.00	0.0634
Spring 6	12.52	17.41	39.40	1.89	7.063	124.50	0.0568
Spring 7	13.52	17.45	39.25	1.93	6.784	125.00	0.0545
Spring 8	12.88	17.51	39.32	1.85	7.781	125.00	0.0625
Spring 9	12.90	17.47	39.41	1.90	7.137	125.00	0.0574
Average	**13.82**	**17.42**	**39.25**	**1.92**	**7.350**	**124.44**	**0.0591**

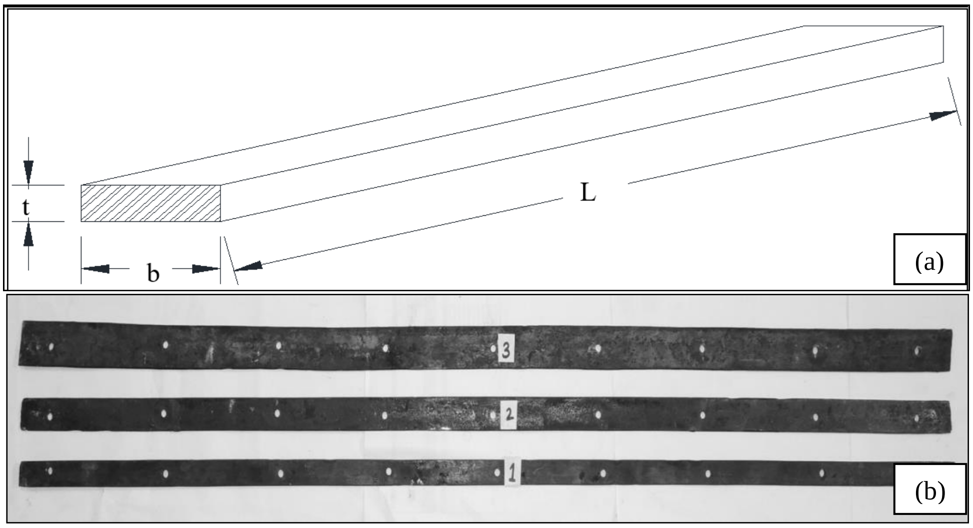

Figure 4. Beam specimen. (a) Schematic diagram (b) Actual beam specimen.

Table 2. Dimensions and mechanical properties of beam specimen.

Beam Specimen	Length L, (m)	Width b, (m)	Thickness t, (m)	Moment of Inertia, I (mm^4)	Modulus of Elasticity, E (N/mm^2)	$\beta = \sqrt[4]{\frac{k}{4EI}}$ (mm^{-1})	βL
1	1.0668	0.20975	0.0033	62.8149	205000	0.0058	6.19
2	1.0668	0.27460	0.0035	98.0312	210000	0.0052	5.55
3	1.0668	0.36340	0.0033	114.419	199000	0.0051	5.44

2.2.1 Test Procedure

In the laboratory, the experimental investigation was carried out using the test setup mentioned earlier. The procedure involved several steps. Firstly, the helical springs were firmly attached to the base beam of the loading frame using connecting bolts. Next, the beam specimen was positioned over the nine equally spaced springs, and small connecting bolts were used to secure the beam to the springs. Deflection gauges were installed at various locations on the beam to measure the vertical deflection at different points.

To apply the load, a loading arrangement was placed at the midpoint of the beam, and five different weights of known values were added to the loading arrangement one by one for each beam. The deflection gauges recorded the vertical deflections at different points on the beam as the each fixed load was applied. This entire process was repeated for all three specimens, presumably to gather data and compare the performance of each specimen.

3 Results and Discussions

This experimental study aims to find the foundation modulus for the developed practical elastic foundation model and the theoretical and actual vertical deflection. To attain these objectives a model of an elastic foundation was fabricated and the vertical deflection at different points are determined for three different beams of different cross sections resting on this foundation. For the determination of foundation modulus, at first the spring constant, K was determined for each spring. Using this foundation modulus, the theoretical deflection values were calculated for different points along each beam and described in this section. Additionally, the experimental test results obtained from the beams on the elastic foundations model are also presented here. These test results are plotted on a graph, with deflection on the vertical axis and the distance of measuring points from the loading point on the horizontal axis. The variations of deflection with respect to the distance from the loading point for different point loads at the center of beam specimens 1, 2, and 3 are shown in Figure 5 (a), (b), and (c) respectively. Figure 5 (d) represents the percentage variation between the actual and theoretical deflection for the different beam specimens.

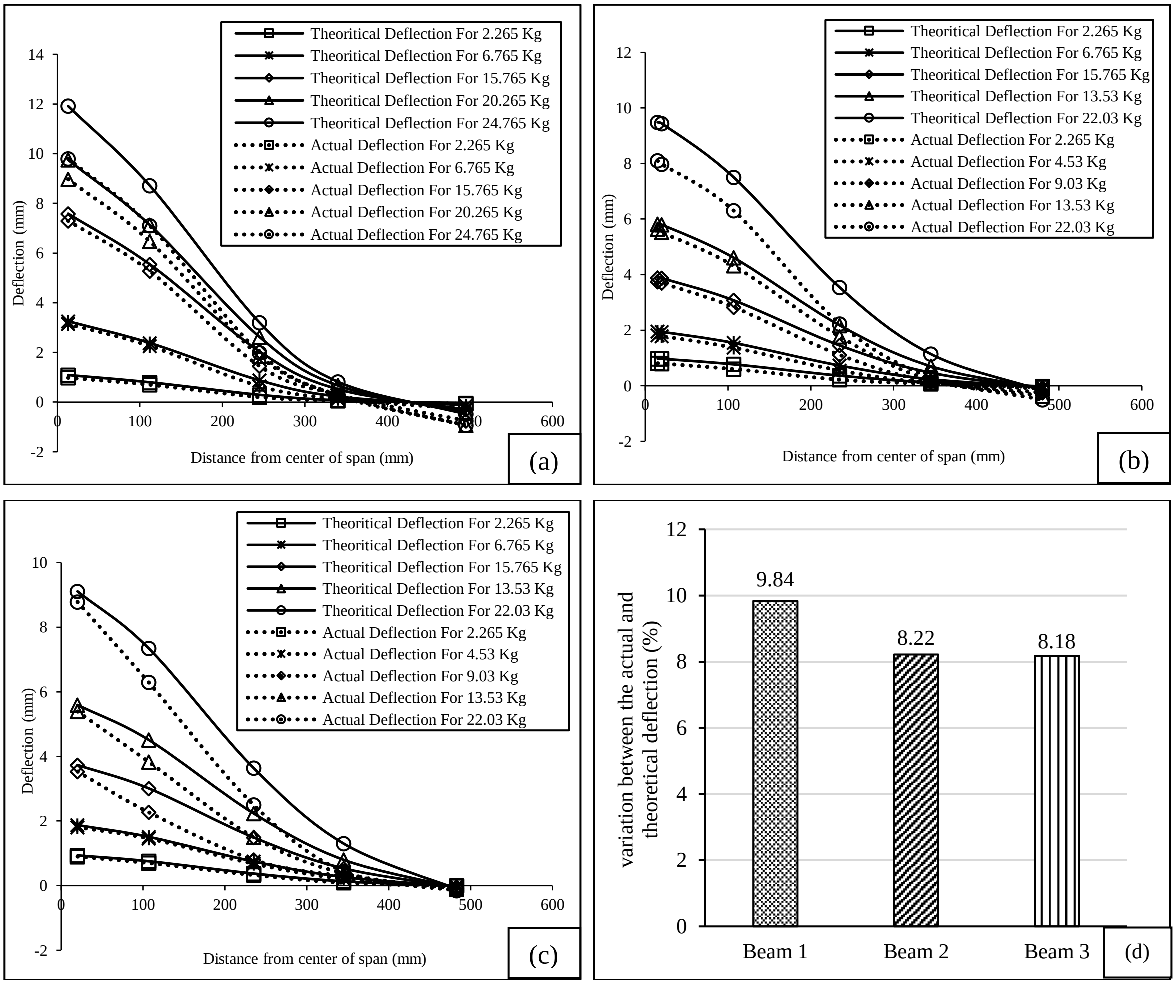

Figure 5. Variations of deflection with respect to the distance from the loading point for different point loads at the center of (a) beam specimen 1 (b) beam specimen 2 (c) beam specimen 3, and (d) percentage variation between the actual and theoretical deflection.

The deflection observed in both the theoretical and experimental cases initially showed a positive value, indicating downward displacement near the loading point. However, after a certain distance, the deflection became negative. It was found that the theoretical and actual zero deflection occurred at the same point for all beam sections, and this point moved further away from the loading point as the beam cross-section increased. In all three beam sections and at every applied load, the actual deflection was slightly less than the theoretical estimation. The variation between the experimental and theoretical values differed depending on the applied loads and beam cross-sections.

Upon calculating the average percentage variation for the different beam specimens, the following observations were made. For beam specimen-1, the theoretical deflection exceeded the actual deflection, resulting in an average variation of 9.84%. Similarly, for beam specimen-2, the theoretical deflection was also greater than the actual deflection, but with a lower average variation of 8.22% compared to beam specimen-1. For beam specimen-3, the theoretical deflection was again greater than the actual deflection, with an average variation of 8.18% (Table 5.8), which was slightly lower than that observed for beam specimen-1 and 2.

In summary, the experimental and theoretical deflection values exhibited variations influenced by the applied loads and beam cross-sections. The average percentage variations indicated that the theoretical deflection generally overestimated the actual deflection, with the magnitude of variation slightly decreasing as the beam cross-section increased.

4 Conclusions

Researchers have been studying the behavior of beams on elastic foundations for over a century, employing a combination of experimental models and analytical solutions. While the majority of previous studies have focused on analytical approaches, this particular investigation involved fabricating an experimental model to directly measure the practical deflection values and compare them with theoretical predictions. By analyzing the results obtained from both analytical calculations and experimental data, several conclusions can be drawn.

Firstly, Theoretical deflection consistently exceeded the practical deflection in all cases examined. This indicates that the theoretical models tend to overestimate the actual deflection behavior of beams on elastic foundations. Furthermore, the variation between actual and theoretical deflection decreased as the cross-sectional area of the beams increased. This implies that beams with larger cross-sectional areas exhibited a closer alignment between theoretical predictions and practical observations. Additionally, the theoretical and experimental zero deflection points occurred at the same location along the length of the beam. This suggests that the theoretical model accurately captures the point where no deflection occurs. However, his study highlights the long-standing efforts to understand the behavior of beams on elastic foundations, with a particular focus on the comparison between theoretical and practical deflection values. The findings underscore the tendency of theoretical models to overestimate deflection and demonstrate the influence of cross-sectional area on the variation between actual and theoretical deflection. Moreover, the alignment of zero deflection points between the theoretical and experimental results confirms the validity of the theoretical model.

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