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Correlation of Critical State Parameter with Plasticity Index of Soils

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Abstract

The Critical State concept is defined as the experimental behavior of saturated reconstituted clays in the triaxial compression tests in an ideal condition and assumed to apply to undisturbed soils. When a situation of the soil sample reaches the critical state, it experiences unlimited distortion while the effective stress and the volume of the soil remain unchanged. The critical state concept relates the effective stresses and the corresponding specific volume (v =1+e) of clay during shearing under drained or undrained conditions, it thus amalgamates the characteristics of shear strength and deformation. To specify some constitutive models such as the Cam clay model, the following four basic soil parameters related to the critical state of soils are required: Compression index (λ), Swell/Recompression Index (ĸ), M(mu), and N(nu). Again, the critical state pore pressure parameter (Λ) is being used to predict undrained shear strengths for various stress histories and initial stress conditions of soils. Critical state parameters of soil are very complex, variable, and difficult to obtain. In the present study, the author attempts to make a correlation between critical state parameters and the plasticity index of the re-constituted soil samples. For this purpose, five soil samples were collected from five different locations in the Khulna City area of Bangladesh. First, re-constituted samples were prepared by standard procedure and laboratory tests were done to calculate the plasticity index of the respective soil samples. After that, the assessment of critical state soil parameters, λ, ĸ, M, N, and Λ was achieved. For this K_0-consolidation test, a consolidated drained triaxial compression (CID) test under isotropic consolidation in a normally loaded state is performed. Finally, the correlations are established between this critical state parameter and the plasticity index of the respective soil samples. The experimental findings showed that the critical state parameters show a linear variation with the plasticity index. The study reveals that the critical state parameters of the reconstituted soil samples Λ, ĸ, and N increased with the increase of plasticity index, while M and Λ decreased slightly with the increase of plasticity index. Finally, correlation equations are developed and with the help of those equations, critical state parameters of soils can be obtained simply by calculating the plasticity index of the soil samples.

Key Words: *Re-constituted soil sample; plasticity index (PI); Critical state parameters of soils; Correlation equation.*

1. Introduction

Critical state parameters of soil are very complex and variable. They vary from place to place according to the properties of the soils which largely depend upon the properties of the rocks from which they are derived. Further, soil properties depend on the proportion or distribution of the various grain sizes, minerals, fabrics, environment, etc. About 80 years ago, the critical state concept was initially used to develop plasticity models for soils (Drucker et al., 1957, Roscoe et al., 1958, Roscoe and Schofield, 1963, Roscoe & Burland, 1968, Schofield & Wroth, 1968, Hai-Sui et al., 2019). After that, many important features of soil behavior have been effectively described by using elastoplastic models developed based on the critical state concept. The critical state concept, according to Roscoe et al. (1968) relates the effective stresses and the corresponding specific volume (v=1+e) of clay during shearing under drained or undrained conditions, it thus unifies the characteristics of shear strength and deformation. For clay generally, the critical state is the condition in which the clay continues to deform at a constant rate under constant effective stress. When a state of the sample reaches the critical state, it experiences unlimited distortion while the effective stress and the volume of the soil remain unchanged. To specify some constitutive models such as the Cam clay model (Schofield and Wroth, 1968), the following four basic soil parameters are required: λ, ĸ, M(mu), and N(nu). Again critical state pore pressure parameter (Λ) can be used to predict undrained shear strengths for various stress histories and initial stress conditions. The above critical state soil parameters of reconstituted samples are used in this study to investigate whether it is possible to determine parameters specifying a constitutive soil model simply by using the plasticity index. Jardine (1985) discussed the implementing detailed investigations of general stress-strain and strength properties using the impact of samples and it was found that most comprehensive studies invariably employed reconstituted soils. A major advantage of using data from reconstituted soils is homogeneity can be represented. Reconstituted samples of Khulna clay are therefore used to investigate the effects of stress deformation characteristics. K_0 - consolidation of this clay slurry with successive increments of load up to 150 kPa, a reconstituted sample was prepared and used for further test process.

2. Critical State Parameters of Soils

2.1 Critical State of Soils and Critical State Line

For clays generally, the critical state is the condition in which the clay continues to deform at constant volume under constant effective stress. The critical state concept represents the idealized behavior of remolded clays, but it is assumed to apply also to undisturbed clays in a triaxial compression test.

The points of all specimens; when they are sheared to failure, lie on a unique line defined as a critical state line. Its projection on the (q, p) plane is a straight line that passes through the origin having a constant slope M as shown in Figure 1. Where q is the deviator stress and p is the effective mean stress.

2.2 Compression index (λ) and Swell/Recompression Index (κ)

According to Sridharan at el. (2005), the compressibility characteristics of soils form one of the important soil parameters required in design considerations. Compression index, λ, which is the slope of the linear portion of void ratio/specific volume (v) vs. logarithm of effective pressure p (log p)/ln p relationship (Figure . b), is extensively used for settlement determination. To specify some constitutive models such as the Cam clay model (Schofield and Wroth, 1968), a compression index is required.

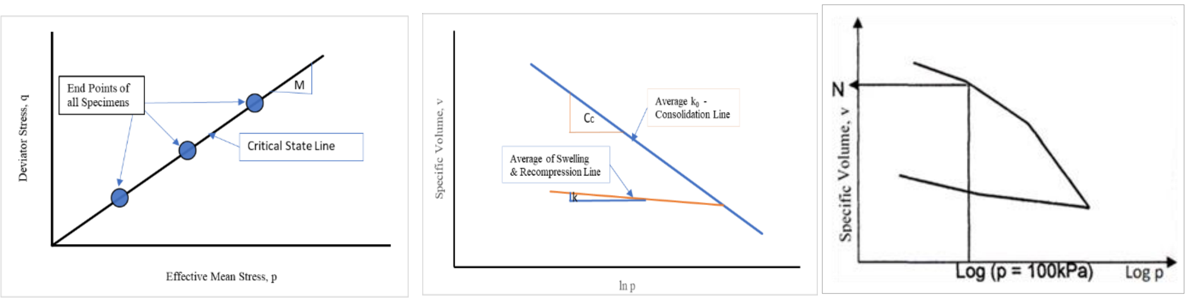
The swell/ Recompression index is also called the free swell index. When the volume of the soil increases without any application of external forces or water pressure, is termed as the free swell. The index will measure the increase in volume concerning the original volume. It is expressed as the slope of the average of swelling and recompression lines. It is obtained from specific volume (v =1 + e) versus In p curve as shown in Figure (b).

2.3 N (nu) and M (mu)

N(nu) is defined as a specific volume form normally consolidation line at p = 100 kPa. It is obtained from specific volume (v =1 + e) versus log p curve as shown in Figure (c). Whereas M(mu) is defined as the slope of the critical state line for isotropic stress conditions. The points of all specimens; when they are sheared to failure, lie on a unique line defined as a critical state line. Its projection on the (q, p) plane is a straight line that passes through the origin having a constant slope, M. Where q is the deviator stress ($q = \sigma_1 - \sigma_3$) and p is the effective mean stress. To specify some constitutive models such as the Cam clay model (Schofield and Wroth, 1968), N and M are required.

2.4 Critical State Pore Pressure Parameter (Λ)

It is important to note that the critical state pore pressure parameter (Λ) has been determined from a total stress approach and it is a soil constant. The parameter A can be used to predict undrained shear strengths for various stress histories and initial stress conditions. The application for the pore pressure parameter is best presented in its relationship with the critical state theory. It is calculated from the formula, $\Lambda = (\lambda - \kappa)/ \lambda$.



a) Critical State Line & M(mu) b) Compression index (λ) & Swell Index(k) C) N(nu)

Figure 1: Critical State line, Compression Index, swell Index, M(mu), N(nu)

3. Preparation of Re-Constituted Soil Samples and Test Process

Disturbed samples were collected from five selected locations in Khulna City, Boyra, Khulna University, Fulbarigate, Shiromony, and near Rupsha Bridge. The soils were taken by excavating up to a depth of about 1.5 to 2m using hand shovels. According to Yin and Miao (2015), the reconstituted soil sample can be prepared by using the following two methods mainly:

i. Preparation by Compaction method and
ii. Preparation by Consolidation of slurry

The second method, preparation by consolidation of slurry is used in this research. The soil slurry was prepared with the five disturbed samples collected from five different locations in the Khulna City area. The samples were first air-dried and then powdered with the help of a hammer. The powdered samples were then sieved, through a No.40 sieve and mixed with water at approximately 1.5 times the liquid limit to form a soil slurry.A consolidation cell of about 152 mm diameter and 305mm height was used for consolidation to form a uniform soil cake from the soil slurry. The soil slurry poured into the K_0-Consolidation cell was first allowed to consolidate by the weight of the porous disc and the self-weight of the sample for 24 hours. Then a pressure of 14 kPa was applied to the sample and this was maintained for 8 hours. In this way, the pressure was gradually increased to about 150 kPa over two days. This pressure was maintained until the primary consolidation is completed, which was indicated by the constant reading of the compression dial gauge. After the completion of the consolidation, the top and bottom parts of the soil cake were extruded by using laboratory tests. Laboratory tests were performed in two different phases

In the first phase, the following index properties are determined by using standard ASTM test procedures: 1) Specific Gravity; 2) Atterberg Limit; 3) Grain Size Distribution (Hydrometer Analysis). In the second phase, critical state parameters are determined by 1) Consolidation Test; 2) Triaxial Test.

4. Test Result and Critical State Parameter Calculation:

Grain size analysis curves of five samples are shown in Figure and the physical properties of five samples are tabulated, is shown in Table 1.

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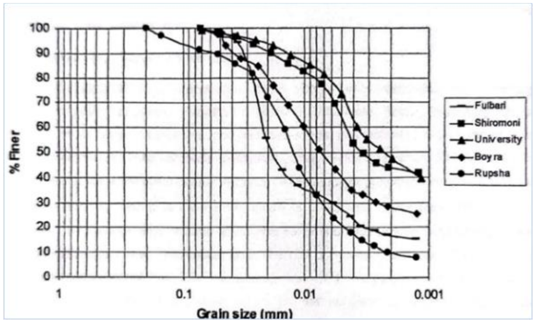


Sp.1= Sample collected from Rupsha bridge site;
Sp.2= Sample collected from Fulbarigate.
Sp.3= SampIe collectedfromBoyra;
Sp.4= SampIe collected from Shiromoni.
Sp.5= Sarnple collected from Khulna University.

Figure 8: Grain Size distribution curve of five samples

Table 3 Index Properties and Classification of the Selected Soil Samples

Parameters	Sp.1	Sp.2	Sp.3	Sp.4	Sp.5
Specific Gravity	2.71	2.70	2.73	2.74	2.73
Liquid Limit (%)	34	38	42	43	58
Plastic Limit (%)	29	28	28	21	35
Plasticity Index (PI)	5	10	14	22	23
Sand (%)	15	3	2	2	8
Silt (%)	67	70	60	37	20
Clay (%)	18	27	38	61	72
USCS Classification	ML	ML	CL	CL	CH

4.1 Critical State Parameters:

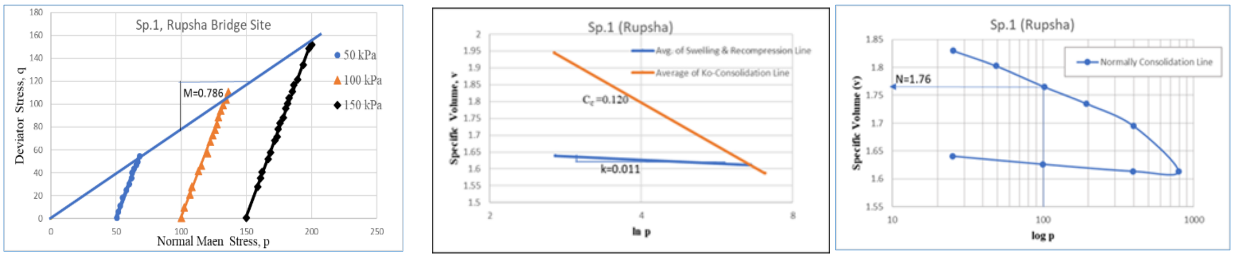
4.1.1 Slope of Critical State Line, M (mu)

The slope of the critical state line, M(mu) is obtained from the stress-strain relationship of the five samples where mean stress, p is plotted on the X-axis and deviator stress, q is plotted on Y-axis for three specimens of a sample of each location under confining pressure 50 kPa, 100 kPa, 150 kPa respectively. The failure points of three specimens lie on a unique line passing through the origin is called a critical state line with a constant slope which is known as critical state parameter M (mu). The values of M (mu) are determined from the p versus q curve for all the samples as shown in Figure 7 (a) and are accumulated in Table 4.

4.1.2 Compression Index, Swell Index, and Critical State Pore Pressure.

The compression index (λ) and swell index(k) are determined from specific volume versus ln p graph where specific volume is void ratio plus one (v = 1 +e) and p was the pressure applied to the reconstituted soil sample during consolidation test. The compression index (λ) is determined from the slope of the average of the K_0-consolidation line. The Swell/Recompression Index (κ) is determined from the slope of the average of swelling & recompression line from the specific volume (v) versus ln p graph. As shown in Figure 7 of sp.1, the Compression Index (Λ) and Swell/Recompression Index (k) of all the samples are determined along with the Critical state pore pressure by using equation $\Lambda = (\lambda - \kappa)/\lambda$ and summarized in Table 4. 4.1.3 N (nu)

It is defined as the value of the specific volume of the normal consolidation line at p = 100 kPa of the specific volume (v =1 + e) versus the log p curve. Where p is the pressure applied to the reconstituted samples during the consolidation test.



a) q vs p of Sp.1 b) Specific Volume (v) vs ln p of Sp.1 b) Specific volume(v) vs log p of
Sp.1

Figure 7: Deviator Stress (q) versus Normal Mean Stress (p) and Specific Volume (v) versus log(p) & ln p curves of sample Sp.1 (Rupsha)

Table 4 Critical State Parameters of the Reconstituted Samples

Parameters Name	**Sp.1**	**Sp.2**	**Sp.3**	**Sp.4**	**Sp.5**
Compression index (λ)	0.120	0.13	0.2	0.192	0.22
Swell Index(k)	0.011	0.016	0.017	0.024	0.025
Critical state Pore Pressure Parameter(Λ)	0.920	0.875	0.916	0.889	0.893
M (mu)	0.786	0.81	0.715	0.687	0.773
N (nu)	1.76	1.77	1.79	1.82	1.83

5. Correlation of the Critical State Parameters with Plasticity Index

Correlations established among different parameters are discussed below:

5.1 Correlation of Plasticity Index (Pl) with Compression Index, and Swell/Recompression Index (κ) :

Figure 8 shows that the compression index (λ), and Swell/Recompression Index (κ) are plotted against the plasticity index (Pl) which reveals, the compressibility of soils increases with an increase in the plasticity index. This trend has been repeatedly reported by various researchers like Sorensen (2015). In their experiments, Kurata and Fujishita (1961) and Akio Nakase, et al. (1988) reported the existence of the linear relationship, deduced from a massive amount of oedometer test data related to the construction of port and harbor facilities. Critical state soil mechanics theory (Schofield and Wroth, 1968) also predicted this relationship, expressed as

$$\Lambda = 0.00585PI \quad \text{………………………………… (1)}$$

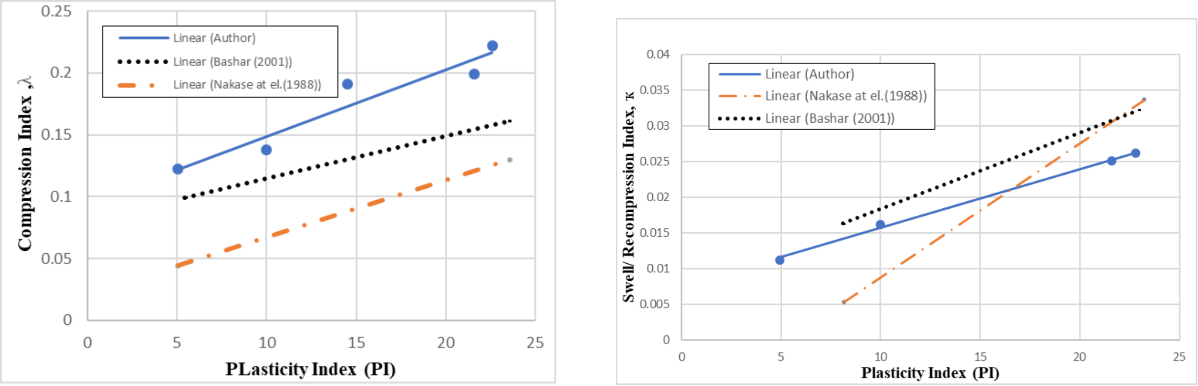
The regression line for the relationship obtained from the present data can be given as

$$\Lambda = 0.0964 + 0.0051Pl \quad \text{…………………….. (2)}$$

A linear relationship is also noticed between the Swell/Recompression Index (ĸ) and plasticity index (Pl) as shown in Figure 8(b) and the same kind of relationship was reported earlier by Hvorslev and Parry (After Schofield and Wroth,1968), AKIO Nakase et al. (1988) and Basher (2001). The regression line for this relationship obtained from the present study can be given as

$$\kappa = 0.0076 + .0051\,\mathrm{Pl} \ldots\ldots\ldots\ldots \quad (3)$$

The values of the coefficient of correlation (R) obtained are remarkably high values of 0.90 to 0.99.



a) Compression index (λ) vs Plasticity Index (PI) b) Swell/Recompression Index (ĸ) vs Plasticity Index (PI)

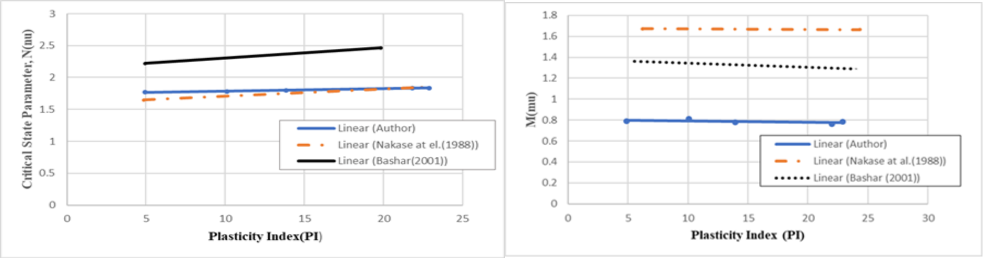
Figure 8: Correlation of Compression index (λ) and Swell/Recompression Index (ĸ) with Plasticity Index (PI)

5.2 Correlation of N (nu) and M (mu) with plasticity index:

Figure 9 (a) illustrates the relationship of N (the values of specific volume at p = 100 kPa, K_0- normally consolidation line) with plasticity index (P1) and Figure 9 (b) shows the relationship of the critical state parameter, M (slope of the critical state line for isotropic stress condition) with Pl. It is observed that there exists a linear relationship of N(nu) and M(mu) with PI. From Figure 9, it is also evident that the values of N increase with the increase of Pl, while the values of M decrease with the increase of Pl. Similar behavior was also reported by Schofild, Wroth (1968), and Bashar (2001). Nakase et al. (1988) reported that N increases with the increase of PI, while M is constant. The resulting relationships found by Nakase et al. (1988) are as follows:

$$N(nu) = 1.5733 + 0.0113\,\mathrm{PI} \ldots\ldots\ldots\ldots \quad (4)$$

$$M(mu) = 1.65 \ldots\ldots\ldots\ldots \quad (5)$$



a) N(nu) versus Plasticity Index (PI) b) M(mu) versus Plasticity Index(PI)

Figure 9: Correlation of N(nu) and M(mu) with Plasticity Index (PI)

Regression lines are obtained from these relationships for the present data can be given as

$$N(nu) = 1.7361 + 0.0039\mathrm{PI} \ldots\ldots\ldots\ldots \quad (6)$$

$$M(mu) = 0.8083 - 0.0037\,\mathrm{PI} \ldots\ldots\ldots\ldots \quad (7)$$

The values of the coefficient of correlation (R) obtained are remarkably high values of 0.93 to 0.99.

Figure 10 illustrates the relationship between the critical state pore pressure parameter (Λ) and plasticity index (Pl). It is found there exists a linear relationship between Λ and PI where the critical state pore pressure parameter increases with the decrease of plasticity index (Pl). Similar behavior was also reported by Parry **(1956)** and Bashar (2001), and the regression line for the relationship obtained by Bashar (2001) is as follows:

Λ= 0.888 - 0.00041 PI……………………………. (8)

The regression line for this relationship obtained from the present data can be given as

Λ = 0.9123 - 0.00061 PI…………………………. (9)

(5.9)

The value of the coefficient of correlation (R) obtained is a remarkably high value of 0.95.

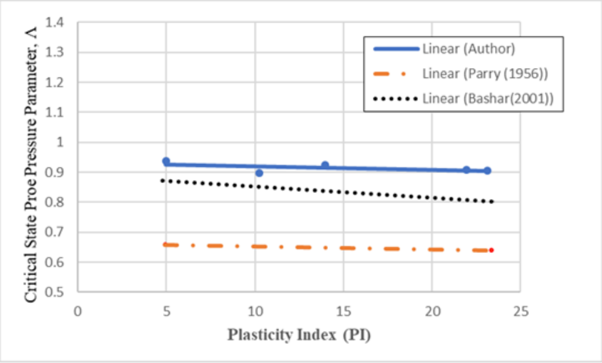


Figure 10: Correlation of Critical State Pore Pressure Parameter (Λ) and Plasticity Index (PI)

6. Conclusion

From finding of this study, it can conclude that the critical state parameters of soils such as compression index (λ), Swell/Recompression Index (κ), critical state pore pressure parameters (Λ), M(mu) and N(nu) have a strong linear correlation with plasticity index (PI). This kind of phenomena are also reported by the previous researchers. Further, the study reveals that the compression index (λ), Swell/Recompression Index (κ), and N(nu) increase whereas critical state pore pressure parameters (Λ) and M(mu) decrease with the increase of plasticity index (PI), are also supported by the findings of the previous study with few exceptions. This few variations are due to variable properties of soils from place to place and critical state parameters also varies with the properties of soils. Critical state parameters are important for understanding the behavior of soils as they are related to the stress-strain, deformation characteristics and strength of soils but very difficult to explore. So, the findings of the study will encourage to evaluate the critical state parameters of soils simply by exploring related index properties of soils like, Plasticity Index.

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